

Prob | Pts

Math 121

Calculus II

Spring 2005

Final Exam
Instructor: _____

Name: (print neatly) _____
(sign) _____

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1. (**10 pts.**) Find the area of the region which is bounded by the graph $y = 1/x$ and the line $x + y = 5/2$.

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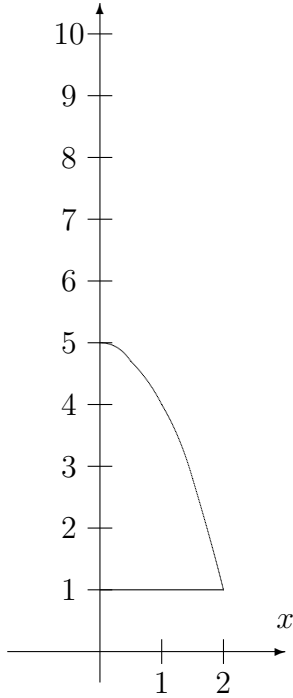
9

2. (**10 pts.**) A function $f(x)$ is defined for $x \geq 1$, and has the property that for every $t > 1$ the area of the region determined by the line $x = 1$, the line $x = t$, the x -axis, and the graph $y = f(x)$ equals e^{1/t^2} . Find $f(x)$.

Total

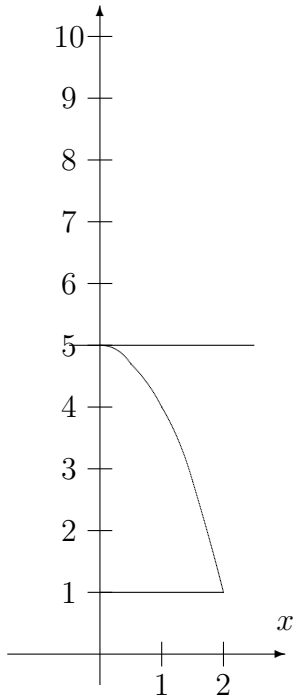
3. (10 pts.) Consider the region bounded by $y = 5 - x^2$, the y -axis and the line $y = 1$, shown. To find the volume of revolution by integrating *with respect to* x , about the y -axis:

- Is the volume element a shell, a disk or a washer?
- What are the dimensions of the element in terms of x ? (E.g. length, radius, etc.)
- What is the integral which gives the volume? [DO NOT EVALUATE THE INTEGRAL]



4. (10 pts.) Consider the region bounded by $y = 5 - x^2$, the y -axis and the line $y = 1$, shown. To find the volume of revolution by integrating *with respect to* x , about the axis $y = 5$:

- Is the volume element a shell, a disk or a washer?
- What are the dimensions of the element in terms of x ? (E.g. length, radius, etc.)
- What is the integral which gives the volume? [DO NOT EVALUATE THE INTEGRAL]



5. (10 pts.) Find the average value of $\ln(x) - 1$ on the interval $[e, e^2]$.

6. (10 pts.) Show that $\int_0^\pi (e^2 + \sin(e^x)) dx > 9$.

7. (40 pts.) Use integration by parts to evaluate the following:

a) $\int x e^{1-x} dx$.

b) $\int (x^3 - x + 1) \ln(x + 1) dx$.

[problem 6 continues on next page]

c) $\int_0^1 x \ln(x^2 + 9) dx.$

d) $\int_1^4 \frac{\ln(x)}{\sqrt{x}} dx.$

8. (40 pts.) Evaluate the following derivatives.

a. $\frac{d}{dx} \ln(xe^x) =$

b. $\frac{d}{dx} \arctan(\sec(x)) =$

c. $\frac{d}{dx} 2^{3x} =$

d) $\frac{d}{dx} \arcsin(1 - x) =$

9. (60 pts.) Compute the following integrals:

a) $\int (x^{\sqrt{2}} + (\sqrt{2})^x + \sqrt{2}^{\sqrt{2}}) dx$

b) $\int \sin(3x - 1) \cos^2(3x - 1) dx$

c) $\int x e^{3x^2} dx$

$$\text{d) } \int \ln(x^3 - x) dx$$

$$\text{e) } \int \sec^3(x) \tan(x) dx$$

$$\text{f) } \int \frac{1 - \ln(x)}{x \ln(x)} dx$$