Name: ____________________________

Mailbox number: ____________________

Math 130 Linear Algebra

Second Test

8 Nov 2006

You may refer to one sheet of notes on this test, and you may use a calculator. You may leave your answers as expressions such as $\binom{8}{4}\frac{e^{1/3}}{\sqrt{2\pi}}$ if you like. Points for each problem are in square brackets.

Problem 1. On cofactor expansion to evaluate determinants [10]

Use cofactor expansion to evaluate the following determinant. (Your choice of which row or column to use.) Show your work.

$$\begin{vmatrix} 1 & 2 & 3 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & -1 & 3 & 0 \\ 5 & 0 & 2 & 3 \end{vmatrix}$$

Problem 2. On orthogonality in $\mathbf{R}^n$ [20]

Prove that if a vector $\mathbf{w}$ is orthogonal to both vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbf{R}^n$, then $\mathbf{w}$ is orthogonal to any linear combination $r\mathbf{u} + s\mathbf{v}$ of them.

Problem 3. On linear transformations [10]

Represent this linear transformation with a matrix.

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y \\ x + z \\ y + z \end{bmatrix}$$

Problem 4. On areas and volumes [20; 10 points each part]

a. Compute the area of the parallelogram whose vertices are located at

$$(2, 3, 1), (4, 5, 2), (6, 3, -1), \text{ and } (8, 5, 0).$$

b. Compute the volume of the parallelepiped with one vertex at the origin, and edges

$$\mathbf{u} = (3, 1, -1), \mathbf{v} = (0, 2, 6), \text{ and } \mathbf{w} = (2, 5, 3).$$

Problem 5. [20] Let $\mathbf{u} = (3, 4, 5)$ and $\mathbf{v} = (2, -1, 0)$. Determine the following.

a. $||\mathbf{u}|| =$

b. $\mathbf{u} \cdot \mathbf{v} =$

c. $\mathbf{u} \times \mathbf{v} =$

d. A unit vector in the same direction as **u**.

e. The cosine of the angle between **u** and **v**.

Problem 6. On abstract vector spaces [20]

Recall the axioms for vector spaces summarized here. A *vector space* is a set equipped with operations of vector addition and scalar multiplication such that

(a) Vector addition is commutative,
(b) Vector addition is associative,
(c) $\mathbf{0}$ is the identity for vector addition,
(d) Each vector has a negation,
(e) Scalar multiplication distributes over vector addition: $c(\mathbf{v} + \mathbf{w}) = c\mathbf{v} + c\mathbf{w}$,
(f) Scalar multiplication distributes over real addition: $(c + d)\mathbf{v} = c\mathbf{v} + d\mathbf{v}$,
(g) Multiplication and scalar multiplication associate: $c(d\mathbf{v}) = (cd)\mathbf{v}$, and
(h) 1 is the identity for scalar multiplication.

Using only these axioms prove that $c\mathbf{0} = \mathbf{0}$ for every scalar c. (You may not assume that the vector space has coordinates, so don't use coordinates for this proof. Work out your proof on the back of one of the test pages, then copy it here when you're satisfied you have it correct. Mention every time your proof uses any axiom. Hint: sometime in this proof you'll need distributivity.)