Math 130 Linear Algebra. Selected answers from section 5.2.

5a. What is the parametric equation of the line through the point $(3, 4, -2)$ parallel to the vector $(4, -5, 2)$?

It's $\mathbf{x} = (3, 4, -2) + t(4, -5, 2)$. Which you could write as $\mathbf{x} = (3+4t, 4-5t, -2+2t)$. You could also write it as three equations

$$\begin{cases} x &= 3+4t \\ y &= 4-5t \\ z &= -2+2t \end{cases}$$

6a. What is the parametric equation of the line through the points $(2, -3, 1)$ and $(4, 2, 5)$?

$\mathbf{x}$ equals the first point plus t times the difference. That is, $\mathbf{x} = (2, -3, 1) + t(2, 5, 4)$. You could also write this in other ways as was done in 5a.

10. Find an equation of the plane passing through the given points.

a. $(0, 1, 2)$, $(3, -2, 5)$, and $(2, 3, 4)$.

In general, the equation is

$$\begin{vmatrix} x & y & z & 1 \\ a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \end{vmatrix} = 0,$$

so a determinant needs to be evaluated and set to 0 get get the equation.

$$\begin{aligned} &\begin{vmatrix} x & y & z & 1 \\ 0 & 1 & 2 & 1 \\ 3 & -2 & 5 & 1 \\ 2 & 3 & 4 & 1 \end{vmatrix} \\ = &\begin{vmatrix} 1 & 2 & 1 \\ -2 & 5 & 1 \\ 3 & 4 & 1 \end{vmatrix} x - \begin{vmatrix} 0 & 2 & 1 \\ 3 & 5 & 1 \\ 2 & 4 & 1 \end{vmatrix} y \\ &+ \begin{vmatrix} 0 & 1 & 1 \\ 3 & -2 & 1 \\ 2 & 3 & 1 \end{vmatrix} z - \begin{vmatrix} 0 & 1 & 2 \\ 3 & -2 & 5 \\ 2 & 3 & 4 \end{vmatrix} \\ = &-12x - 0y + 12z - 24 \end{aligned}$$

Therefore, the equation is $-x + z = 2$. It's easy to check that this equation is, as it should be, satisfied by the three points.

b. $(2, 3, 4)$, $(-1, -2, 3)$, and $(-5, -4, 2)$.

$$\begin{aligned} &\begin{vmatrix} x & y & z & 1 \\ 2 & 3 & 4 & 1 \\ -1 & -2 & 3 & 1 \\ -5 & -4 & 2 & 1 \end{vmatrix} \\ = &\begin{vmatrix} 3 & 4 & 1 \\ -2 & 3 & 1 \\ -4 & 2 & 1 \end{vmatrix} x - \begin{vmatrix} 2 & 4 & 1 \\ -1 & 3 & 1 \\ -5 & 2 & 1 \end{vmatrix} y \\ &+ \begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ -5 & -4 & 1 \end{vmatrix} z - \begin{vmatrix} 2 & 3 & 4 \\ -1 & -2 & 3 \\ -5 & -4 & 2 \end{vmatrix} \\ = &\ 3x + 1y - 14z + 47 \end{aligned}$$

Therefore, the equation is $3x + y - 14z = -47$.

13. Are the points $\mathbf{u} = (2, 3, -1)$, $\mathbf{v} = (4, -2, -3)$, and $\mathbf{w} = (0, 8, -1)$ on the same line in 3-space?

An equivalent question is: are the differences parallel? It's enough to take any two of the differences, say, $\mathbf{u} - \mathbf{v} = (-2, 5, 1)$ and $\mathbf{v} - \mathbf{w} = (4, -2, -3)$. Since the second is -2 times the first, they're parallel.

Note that the triple product $[\mathbf{u}, \mathbf{v}, \mathbf{w}]$ is 0 if and only if $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ lie on the same plane through the origin $\mathbf{0}$, and that's a weaker condition. So checking to see if the triple product is 0 is not enough.

T.3. The lines L_1 and L_2 in $\mathbf{R}^3$ are said to be *skew* if they are not parallel and do not intersect. Give an example of skew lines L_1 and L_2.

Almost any pair of lines are skew. Here's one pair that are skew. Let L_1 be the x-axis, and let L_2 be the line 1 unit above the y-axis. If you like, you can describe the lines parametrically. Here are descriptions of these two lines given parametrically.

$$L_1 : (0, 0, 0) + t(1, 0, 0)$$

$$L_2 : (0, 0, 1) + t(0, 1, 0)$$

Note: a single equation in x, y, and z does not describe a line in $\mathbf{R}^3$; it describes a plane in $\mathbf{R}^3$. Two equations are needed to describe a line since two planes intersect in a line (unless, of course, the planes are parallel or the same plane).